OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4721/01**

MATHEMATICS

Core Mathematics 1

THURSDAY 7 JUNE 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

WARNING

You are not allowed to use a calculator in this paper.

This document consists of **4** printed pages.

1 Simplify $(2x + 5)^2 - (x - 3)^2$, giving your answer in the form $ax^2 + bx + c$. [3]

2 **(a)** On separate diagrams, sketch the graphs of

(i) $y = \frac{1}{x}$, [2]

(ii) $y = x^4$. [1]

(b) Describe a transformation that transforms the curve $y = x^3$ to the curve $y = 8x^3$. [2]

3 Simplify the following, expressing each answer in the form $a\sqrt{5}$.

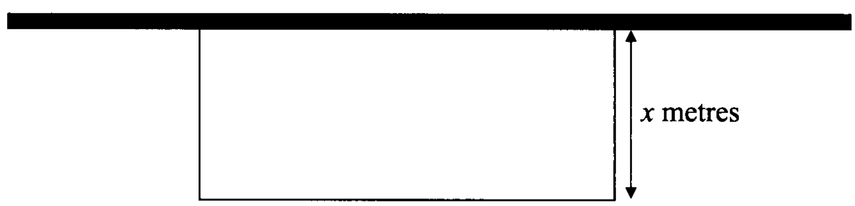
(i) $3\sqrt{10} \times \sqrt{2}$ [2]

(ii) $\sqrt{500} + \sqrt{125}$ [3]

4 **(i)** Find the discriminant of $kx^2 - 4x + k$ in terms of k. [2]

(ii) The quadratic equation $kx^2 - 4x + k = 0$ has equal roots. Find the possible values of k. [3]

5



The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres.

(i) Show that the enclosed area, A m^2, is given by

$$A = 20x - 2x^2.$$ [2]

(ii) Use differentiation to find the maximum value of A. [4]

6 By using the substitution $y = (x + 2)^2$, find the real roots of the equation

$$(x + 2)^4 + 5(x + 2)^2 - 6 = 0.$$ [6]

7 **(a)** Given that $f(x) = x + \frac{3}{x}$, find $f'(x)$. [4]

(b) Find the gradient of the curve $y = x^{\frac{5}{2}}$ at the point where $x = 4$. [5]

8 **(i)** Express $x^2 + 8x + 15$ in the form $(x + a)^2 - b$. [3]

(ii) Hence state the coordinates of the vertex of the curve $y = x^2 + 8x + 15$. [2]

(iii) Solve the inequality $x^2 + 8x + 15 > 0$. [4]

9 The circle with equation $x^2 + y^2 - 6x - k = 0$ has radius 4.

(i) Find the centre of the circle and the value of k. [4]

The points $A\ (3,\ a)$ and $B\ (-1,\ 0)$ lie on the circumference of the circle, with $a > 0$.

(ii) Calculate the length of AB, giving your answer in simplified surd form. [5]

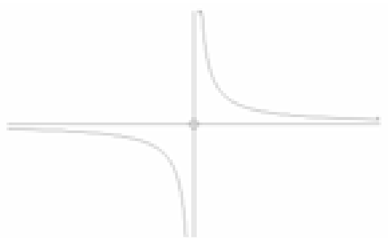
(iii) Find an equation for the line AB. [3]

10 **(i)** Solve the equation $3x^2 - 14x - 5 = 0$. [3]

A curve has equation $y = 3x^2 - 14x - 5$.

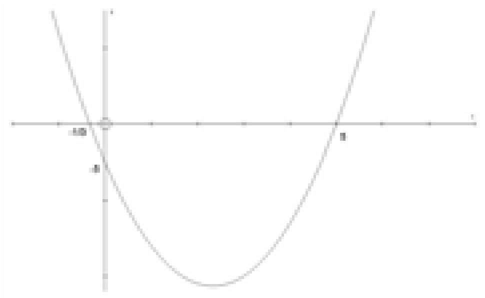
(ii) Sketch the curve, indicating the coordinates of all intercepts with the axes. [3]

(iii) Find the value of c for which the line $y = 4x + c$ is a tangent to the curve. [6]

1	$(4x^2 + 20x + 25) - (x^2 - 6x + 9)$ $= 3x^2 + 26x + 16$ <u>Alternative method using difference of two squares:</u> $(2x + 5 + (x - 3))(2x + 5 - (x - 3))$ $= (3x + 2)(x + 8)$ $= 3x^2 + 26x + 16$	M1 A1 A1 3 **3**	Square one bracket to give an expression of the form $ax^2 + bx + c$ $(a \neq 0,\ b \neq 0,\ c \neq 0)$ One squared bracket fully correct All 3 terms of final answer correct M1 2 brackets with same terms but different signs A1 One bracket correctly simplified A1 All 3 terms of final answer correct
2 (a)(i) (ii) (b)	 Stretch Scale factor 8 in y direction **or** scale factor ½ in x direction	B1 B1 2 B1 1 B1 B1 2 **5**	Excellent curve for $\frac{1}{x}$ in either quadrant Excellent curve for $\frac{1}{x}$ in other quadrant **SR B1** Reasonably correct curves in 1st and 3rd quadrants Correct graph, minimum point at origin, symmetrical
3 (i) (ii)	$3\sqrt{20}$ or $3\sqrt{2}\ \sqrt{5} \times \sqrt{2}$ or $\sqrt{180}$ or $\sqrt{90} \times \sqrt{2}$ $= 6\sqrt{5}$ $10\sqrt{5} + 5\sqrt{5}$ $= 15\sqrt{5}$	M1 A1 2 M1 B1 A1 3 **5**	 Correctly simplified answer Attempt to change both surds to $\sqrt{5}$ One part correct and fully simplified cao

4 (i)	$(-4)^2 - 4 \times k \times k$ $= 16 - 4k^2$	M1 A1 2	Uses $b^2 - 4ac$ (involving k) $16 - 4k^2$
(ii)	$16 - 4k^2 = 0$ $k^2 = 4$ $k = 2$ or $k = -2$	M1 B1 B1 3 **5**	Attempts $b^2 - 4ac = 0$ (involving k) or attempts to complete square (involving k)
5 (i)	Length = 20 – 2x Area = x(20 – 2x) = $20x - 2x^2$	M1 A1 2	Expression for length of enclosure in terms of x Correctly shows that area = $20x - 2x^2$ **AG**
(ii)	$\frac{dA}{dx} = 20 - 4x$ For max, 20 – 4x = 0 x = 5 only Area = 50	M1 M1 A1 A1 4 **6**	Differentiates area expression Uses $\frac{dy}{dx} = 0$
6	Let $y = (x + 2)^2$ $y^2 + 5y - 6 = 0$ $(y + 6)(y - 1) = 0$ y = -6 or y = 1 $(x + 2)^2 = 1$ x = -1 or x = -3	B1 M1 A1 M1 A1 A1 6 **6**	Substitute for $(x + 2)^2$ to get $y^2 + 5y - 6$ (= 0) Correct method to find roots Both values for y correct Attempt to work out x One correct value Second correct value and no extra real values
7 (a)	$f(x) = x + 3x^{-1}$ $f'(x) = 1 - 3x^{-2}$	M1 A1 A1 A1 4	Attempt to differentiate First term correct x^{-2} soi **www** Fully correct answer
(b)	$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$ When x = 4, $\frac{dy}{dx} = \frac{5}{2}\sqrt{4^3}$ = 20	M1 B1 B1 M1 A1 5 **9**	Use of differentiation to find gradient $\frac{5}{2}x^c$ $kx^{\frac{3}{2}}$ $\sqrt{4^3}$ soi **SR** If 0 scored for first 3 marks, award B1 if $\sqrt{4^n}$ correctly evaluated.

8 (i)	$(x + 4)^2 – 16 + 15$ $= (x + 4)^2 – 1$	B1 M1 A1 3	a = 4 15 – their a^2 cao in required form
(ii)	(-4, -1)	B1 ft B1 ft 2	Correct x coordinate Correct y coordinate
		M1 A1	Correct method to find roots -5, -3
(iii)	$x^2 + 8x + 15 > 0$ $(x + 5)(x + 3) > 0$ $x < -5, x > -3$	M1 A1 4 **9**	Correct method to solve quadratic inequality eg +ve quadratic graph x < -5, x > -3 (not wrapped, strict inequalities, no 'and')
9 (i)	$(x - 3)^2 – 9 + y^2 - k = 0$ $(x - 3)^2 + y^2 = 9 + k$ Centre (3, 0) $9 + k = 4^2$ $k = 7$	B1 B1 M1 A1 4	$(x-3)^2$ soi Correct centre Correct value for *k* (may be embedded) <u>Alternative method using expanded form:</u> Centre (-*g*, -*f*) M1 Centre (3, 0) A1 $4 = \sqrt{f^2 + g^2 - (-k)}$ M1 *k* = 7 A1
(ii)	$(3 - 3)^2 + y^2 = 16$ $y^2 = 16$ y = 4 Length of AB = $\sqrt{(-1-3)^2 + (0-4)^2}$ $= \sqrt{32}$ $= 4\sqrt{2}$	M1 A1 M1 A1 ft A1 5	Attempt to substitute x = 3 into original equation or their equation y = 4 (do not allow ± 4) Correct method to find line length using Pythagoras' theorem $\sqrt{32}$ or $\sqrt{16+a^2}$ cao
(iii)	Gradient of AB = 1 or $\frac{a}{4}$ y – 0 = m(x + 1) or y – 4 = m (x – 3) y = x + 1	B1 ft M1 A1 3 **12**	 Attempts equation of straight line through their A or B with their gradient Correct equation in any form with simplified constants

10 (i)	$(3x + 1)(x - 5) = 0$	M1	Correct method to find roots
	$x = \frac{-1}{3}$ or $x = 5$	A1	Correct brackets or formula
		A1 3	Both values correct
			SR B1 for x = 5 spotted **www**
(ii)		B1	Positive quadratic (must be reasonably symmetrical)
		B1	y intercept correct
		B1 ft 3	both x intercepts correct
(iii)	$\frac{dy}{dx} = 6x - 14$	M1*	Use of differentiation to find gradient of curve
	$6x - 14 = 4$	M1*	Equating their gradient expression to 4
	$x = 3$	A1	
	On curve, when x = 3, y = -20	A1 ft	Finding y co ordinate for their x value
	$-20 = (4 \times 3) + c$	M1dep	N.B. dependent on both previous M marks
	$c = -32$	A1 6	
	Alternative method:		
	$3x^2 - 14x - 5 = 4x + c$	M1	Equate curve and line (or substitute for x)
	$3x^2 - 18x - 5 - c = 0$ has one solution	B1	Statement that only one solution for a tangent (may be implied by next line)
	$b^2 - 4ac = 0$	M1	Use of discriminant = 0
	$(-18)^2 - (4 \times 3 \times (-5 - c)) = 0$	M1	Attempt to use a, b, c from their equation
	$c = -32$	A1	Correct equation
		A1	c = -32
		12	